Interpolating a function between two known values, if it can be assumed that the function can be simulated by general exponential decay.

Given

- We know, or can measure, the values of the function at two points, t_1 and t_2
- We can make the assumption that the function follows exponential decay of the type $F(t) = ae^{-bt}$ (between the two points t_1 and t_2)
- We want to evaluate the function at some time $t=t_{1-2}$ (between t_1 and t_2)

Solution

The function at the two points, t_1 and t_2, can be written as:

$$F_1 = ae^{-bt_1} \quad (1) \quad \text{and} \quad F_2 = ae^{-bt_2} \quad (2)$$

Dividing F_1 by F_2 we have,

$$\frac{F_1}{F_2} = \frac{ae^{-bt_1}}{ae^{-bt_2}} = e^{-bt_1-(-bt_2)} = e^{b(t_2-t_1)} \quad (3)$$

Solving this equation for *b* we get,

$$\frac{F_1}{F_2} = e^{b(t_2-t_1)}, \ \ln\left[\frac{F_1}{F_2}\right] = b(t_2-t_1), \ b = \frac{\ln\left[\frac{F_1}{F_2}\right]}{(t_2-t_1)} \quad (4)$$

Using the original equation (1) for F_1, and solving for *a* we have

$$a = \frac{F_1}{e^{-bt_1}} = F_1 e^{bt_1} = F_1 e^{\frac{\ln\left[\frac{F_1}{F_2}\right]}{(t_2-t_1)}t_1} \quad (5)$$

Now, knowing *a* and *b*, for a *t* in the domain $t_1 < t < t_2$ we can get *F(t)*

$$F_t = ae^{-bt} = \left(F_1 e^{\frac{\ln\left[\frac{F_1}{F_2}\right]}{(t_2-t_1)}t_1}\right) e^{-\left\{\frac{\ln\left[\frac{F_1}{F_2}\right]}{(t_2-t_1)}\right\}t} = F_1 e^{\left\{\frac{\ln\left[\frac{F_1}{F_2}\right]}{(t_2-t_1)}\right\}(t_1-t)} \quad (6)$$

Validation

A numerical validation test was set-up using a spreadsheet, as shown below, using a known analytical exponential function $F(t) = 1.25e^{-0.675}$ The analytical values of the function were calculated for *t=1* to *t=13* and then the Regression values at the midpoint of each *t* interval were calculated with the equations derived here. The results were also plotted to demonstrate the ability of the regression formulas to predict *F(t)* values at other t.

	A	B	C	D	E
1					
2	Analytical Model Using				
3	F(t)=a*EXP(-b*t)			Regression Model	
4	a =	1.25		Using the Derived formulas	
5	b =	0.675		For example, for cell E8 Using	
6	t	F(t)		=B7*EXP((LN(B7/B8)/(A8-A7))*(A7-D8))	
7	1	0.636445526		t	F (t)
8	2	0.324050326		1.5	0.454136962
9	3	0.164992304		2.5	0.23122675
10	4	0.084006891		3.5	0.117730584
11	5	0.042772648		4.5	0.059943283
12	6	0.021777968		5.5	0.030520507
13	7	0.011088392		6.5	0.015539712
14	8	0.005645726		7.5	0.007912144
15	9	0.002874558		8.5	0.004028519
16	10	0.0014636		9.5	0.002051146
17	11	0.000745201		10.5	0.001044354
18	12	0.000379424		11.5	0.00053174
19	13	0.000193186		12.5	0.000270739